Testing and Modeling a Physical Galton Board

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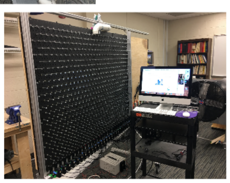
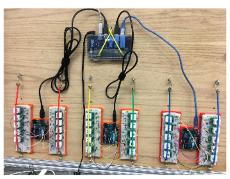
Abstract

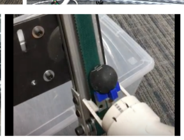
In this project we created a physical model of a Galton board where the ball's path is determined by physical collisions with each peg and by actual physical parameters of the ball, pegs, and board. To test the model, we built a Galton board and compared results with the computational model. The model produced similar distributions of drift time and final bin position. However, the statistical variation in the model completely depended on random variation in the initial position of the ball, and the statistical variation in the experiment depended on the apparent variation of physical parameters of the ball and board (because its initial position was nearly constant). The computational model is similar enough to the actual board that the model can be used to investigate how the statistical distribution of the ball depends on various parameters. The Galton board is an instrument that involves statistics and many fundamental and classical physics concepts such as collisions, angular momentum, Newton's Second Law of Motion and Conservation of Energy. Thus, the board has educational value and can also be used as an analogy for DC current in a wire.

Apparatus

The area of the pegboard is 4 ft high x 6 ft wide. Pegs are 0.25 in bolts, 2 inches apart. Rows are spaced 2 inches apart. There are 28 laser gates, one at each bin, to detect a ball falling into a particular bin. The laser gates were monitored by Arduinos. The Arduinos sent the data to a Python program which produced a histogram of ball count vs. bin at Plotly. The apparatus was automated with a conveyor belt to continually drop balls through the apparatus in order to gain sufficient data.

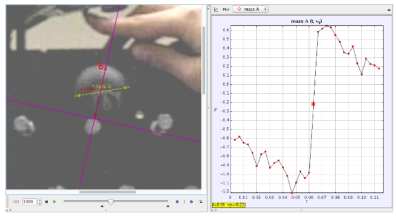




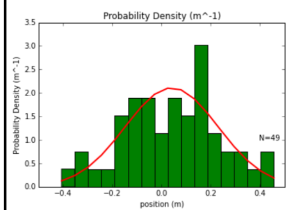


Experiment 1

Video analysis was used to measure the physical parameters for a collision of a squash ball and peg as described by Cross(2005). The parameters were used in the computational model.



Experiment 2

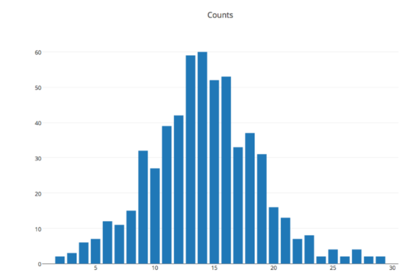


The ball was manually released from rest using a clamp at a position between two pegs on the top row. For 49 trials, the drift time and final bin position were recorded. Results were:

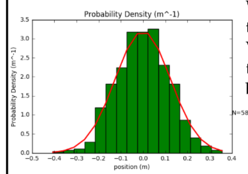
$$\bar{t}_{drift} = 5.5\,\mathrm{s} \pm 0.3\,\mathrm{s}$$
$$\bar{v}_{drift} = 0.20\,\mathrm{m/s} \pm 0.01\,\mathrm{m/s}$$
$$\bar{x}_{bin} = 0.04\,\mathrm{m} \pm 0.19\,\mathrm{m}$$

Experiment 3

In this experiment, the belt and laser gates were added in order to run the apparatus for a long time and automatically compute the histogram. We ran it for 9 hours and produced the following histogram. ($N = 581, x_{bin} = 0.0049 \pm 0.59\,\mathrm{m}$)



Computational Model

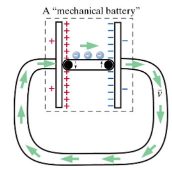


We created a physical computational model of our board in VPython with the same specifications and dimensions as the actual board. Results were ($N = 581$):

$$\bar{t}_{drift} = 6.1\,\mathrm{s} \pm 0.3\,\mathrm{s}$$
$$\bar{v}_{drift} = 0.18\,\mathrm{m/s} \pm 0.01\,\mathrm{m/s}$$
$$\bar{x}_{bin} = -0.00096 \pm 0.12\,\mathrm{m}$$

The bin distribution had less variance than Experiment 3 The drift time was greater and drift speed was less than in Experiment 2. This is partially explained by the one radius increase in the initial y position and the additional collisions with pegs in the top row.

Applications



1. **DC Circuit demonstration.** The board can be used as an analogy for the Drude model. $v_d = uE$. Peg density and ball diameter affect mobility. Tilting the board is like changing electric field. The conveyor belt is like a battery. (Image: *Matter and Interactions*, 4th ed. by Chabay and Sherwood.)
2. **SPS public outreach events.** Kids drop a ball and predict which bin it will go through.

References

- Kozlov, V. V. and Mitrofanova, M. Yu. "Galton Board." *Regular Chaotic Dynamics* 8 (2002), 431-439.
- Christian. W. "Galton Board JS Model." http://www.opensourcephysics.org/items/detail.cfm?ID=13383. (2014).
- R. Cross, "Bounce of a spinning ball near normal incidence," Am. J. Phys. 73, 914–920 (2005).

Acknowledgments

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